CP GEOMETRY

Rigid Motion with Parallel Lines and Congruent Triangles

Unit 1B-Rigid Motion with Parallel Lines and Congruent Triangles

Learning Targets

1.6 Angle Pairs

17. I can use angle pair relationships to find unknown values or angle measures.
    a. Adjacent
    b. Linear Pair
    c. Supplementary
    d. Complementary
    e. Vertical angles

18. I can use angle pair relationships formed by parallel lines cut by a transversal.
    a. Corresponding Angles
    b. Alternate Interior Angles
    c. Alternate Exterior Angles
    d. Same Sided Interior Angles
    e. Same Sided Exterior Angles

1.7 Transformations to prove congruency

19. I can demonstrate using rigid motions that two figures are congruent by 1:1mapping.

1.8 Congruent Triangles

20. I can write a congruency statement of congruent figures.
21. I can identify corresponding parts of congruent figures.
22. I can determine if two triangles are congruent by SSS, SAS, ASA, AAS, and HL using given markings and assumptions.
23. I can prove two triangles are congruent by SSS, SAS, ASA, AAS, and HL using givens, assumptions, and theorems.
24. I can use CPCTC in a proof.

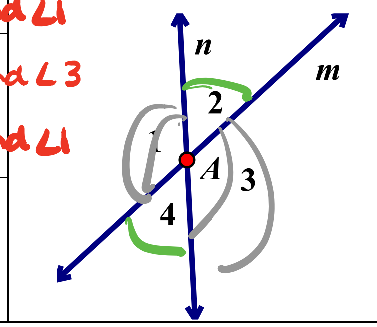
1.9 Triangle Properties

25. I can solve for unknowns using the sum of the interior angles of a triangle is 180°.
26. I can solve for unknowns using the base angles of an isosceles triangle are congruent.
27. I can solve for unknowns using the exterior angle theorem.
28. I can solve for unknowns using the mid-segment (midline) of triangle theorem.

Geometry
1.6(1) Angle Pairs

Objective: I can use angle pair relationships to find unknown values or angle measures *(Target 17)*

When two lines intersect in a plane, a number of angles are formed. In the diagram below, the intersection of line m and line n is point A. There exist different types of angle pairs with various relationships.

ADJACENT ANGLES	Two angles that share a vertex and a ray (side).	∠1 and ∠2 ∠2 and ∠3 ∠3 and ∠4 ∠4 and ∠1	
LINEAR PAIR	Two angles that are adjacent whose measure add up to 180° (non-shared sides form opposite rays).	∠1 and ∠2 ∠2 and ∠3 ∠3 and ∠4 ∠4 and ∠1	
VERTICAL ANGLES	Two angles that are non-adjacent but share a vertex and sides form opposite rays. congruent	∠1 and ∠3 ∠2 and ∠4	

1. Determine whether the angles are VERTICAL ANGLES, a LINEAR PAIR, ADJACENT ANGLES, or NEITHER.

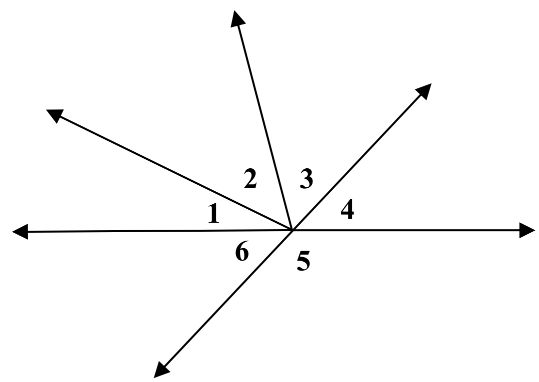
∠4 and ∠5 linear pair

∠3 and ∠6 none of the above

∠6 and ∠4 vertical angles

∠2 and ∠5 none of the above

∠1 and ∠2 adjacent

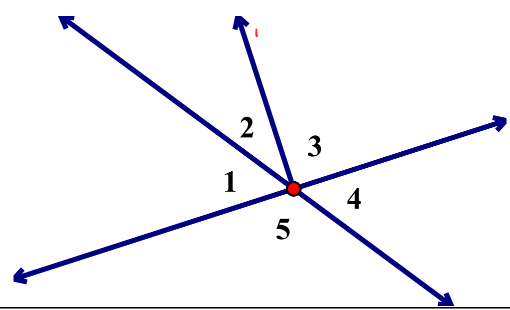


2. Determine if the following statements are True or False.

∠4 and ∠3 are adjacent angles. T or F

∠4 and ∠1 are vertical angles. T or F

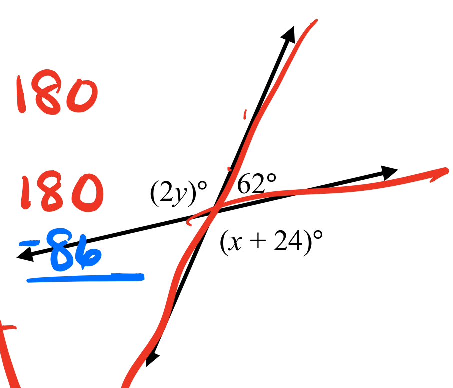
∠3 and ∠4 are a linear pair. T or F



3. Solve for *x* and *y*.

$2y + \cancel{62} = 180$
$\quad -\cancel{62} \quad -62$
$\cancel{2}y = 118$
$\quad \cancel{2} \quad\quad 2$
$\boxed{y = 59}$

$\underbrace{62 + x + 24}= 180$
$x + \cancel{86} = 180$
$\quad -\cancel{86} \quad -86$
$\boxed{x = 94}$

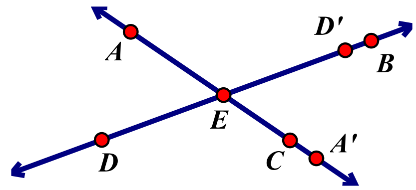


To prove something is to logically establish connections from what you know to what you want to prove while providing accurate reasoning for each conclusion. You must explain ***what*** *you know and* ***why*** *you know it.*

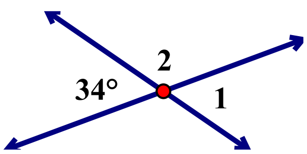
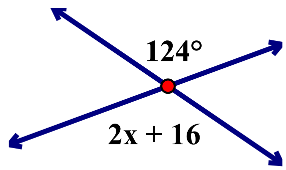
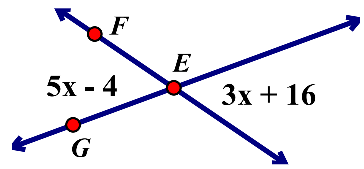
VERTICAL ANGLES

Prove the Vertical Angles Theorem …

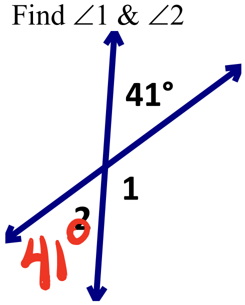
Prove: $\angle DEA \cong \angle BEC$



1. ***D*** rotates 180° around point *E* and maps to ***D'*** on opposite ray $\overrightarrow{EB}$.
2. Similarly, ***A*** rotates 180° around point *E* and maps to ***A'*** on opposite ray $\overrightarrow{EC}$.
3. Now $\angle D'EA' \cong \angle BEC$ because the angles use the same rays and vertex.

Find m∠1 and m∠2	Solve for *x*	Find m∠FEG
 **m∠1 = 34°** (vertical ∠s ≅) **m∠1** + m∠2 = 180° (linear pair) **34°** + m∠2 = 180° (substitution) **m∠2 = 180 – 34 = 156°**	 2x + 16 = 124 (vertical ∠s ≅) 2x = 108 **x = 54**	 5x – 4 = 3x + 16 (vertical ∠s ≅) 2x = 20 x = 10 5(10) – 4 = **46° = m∠FEG**

4. a) Find ∠1 & ∠2

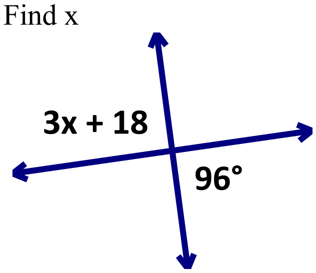


$\boxed{m\angle 2 = 41°}$

180 - 41 = 139

$\boxed{m\angle 1 = 139°}$

b) Find x

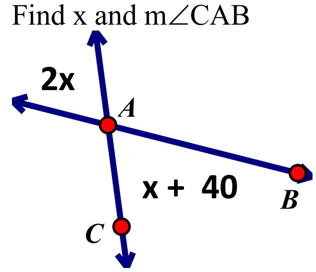


$3x + \cancel{18} = 96$
$\cancel{-18} \quad -18$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{78}{3}$

$\boxed{x = 26}$

c) Find x and m∠CAB



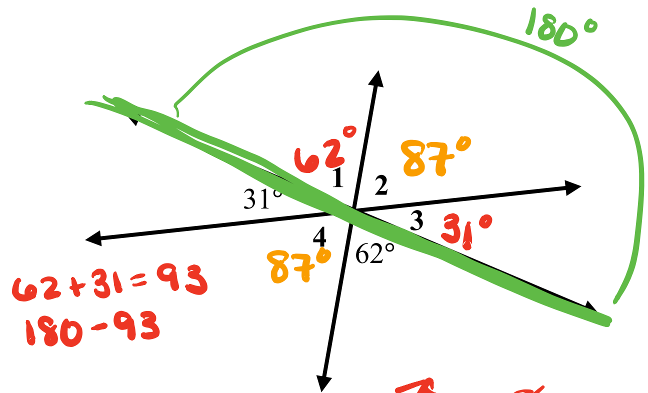
$2x = \cancel{x} + 40$
$-x \quad \cancel{-x}$

$\boxed{x = 40}$

5. Use the diagram to find all missing angles.

m∠1 = 62°   m∠2 = 87°

m∠3 = 31°   m∠4 = 87°



SUPPLEMENTARY 8 **ANGLES**	A pair of angles whose sum of measures is 180°. *(they often form a straight line)*	90° 81°
COMPLEMENTARY 9 **ANGLES**	A pair of angles whose sum of measures is 90°. *(they often form a right angle)*	41° 49°

6. Given the diagram, find the following. Use correct notation.

A linear pair: ∠ENS and ∠ENA

Another linear pair: ∠LNA and ∠LNG

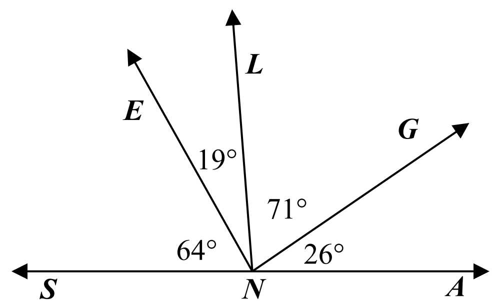
adds to 180° with

The supplement to ∠*ENS*: ∠ENA

adds to 90° with

The complement to ∠*ENL*: ∠LNG

what makes 90° with 19°



7. Complete each statement using the diagram shown.

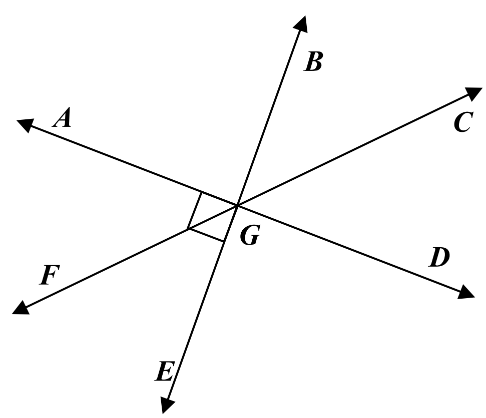
∠*AGB* and ∠EGD are vertical angles.

∠*BGC* and ∠BGF form a linear pair.

∠*FGD* is supplementary to ∠CDG.

∠*EGD* and ∠DGC are adjacent.

∠*BGC* is complementary to ∠CGD.

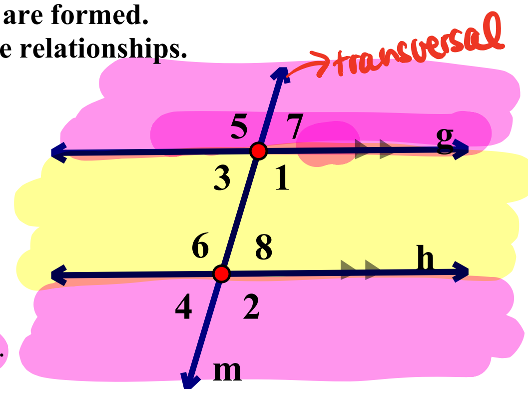


Geometry

1.6(2) Angle Pairs

Objective: I can determine the relationships of angles formed by two parallel lines cut by a *transversal (Target 19)*

When a line intersects two parallel lines, EIGHT angles are formed.
We are now going to learn about the different pairs of angle relationships.



The line that intersects the parallel lines is called the **TRANSVERSAL.**

What is the transversal in the diagram shown? line m

The angles whose interior points lie BETWEEN the parallel lines are **INTERIOR**.

Which angles are interior in the diagram shown? ∠1, ∠3, ∠6, ∠8

The angles whose interior points lie OUTSIDE of the parallel lines are **EXTERIOR**.

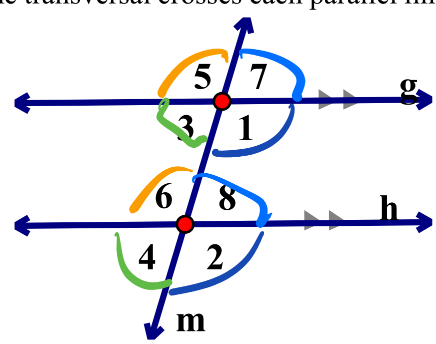
Which angles are exterior in the diagram shown? ∠2, ∠4, ∠5, ∠7

CORRESPONDING ANGLES

A pair of angles that have the same relative position at both intersections where the transversal crosses each parallel line are called CORRESPONDING ANGLES.

What are the corresponding angle pairs in the diagram?

∠7 and ∠8
∠3 and ∠4
∠5 and ∠6
∠2 and ∠1



ALTERNATE INTERIOR and EXTERIOR ANGLES

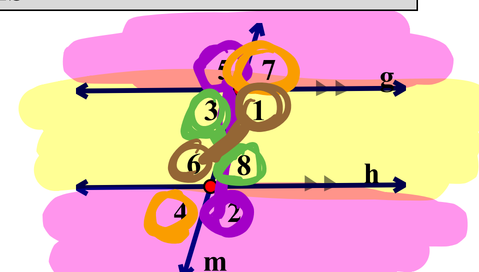
A pair of angles that lie on alternating sides of the transversal.

What are the alternate exterior angle pairs in the diagram?

∠2 and ∠5   ∠4 and ∠7

What are the alternate interior angle pairs in the diagram?

∠3 and ∠8   ∠1 and ∠6



SAME SIDE (CONSECUTIVE) INTERIOR and EXTERIOR ANGLES

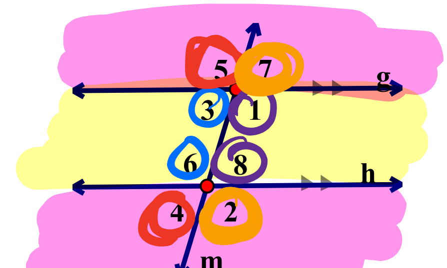
A pair of angles that lie on the same sides of the transversal.

What are the same side (consecutive) interior angle pairs in the diagram?

∠3 and ∠6   ∠1 and ∠8

What are the same side (consecutive) exterior angle pairs in the diagram?

∠4 and ∠5   ∠2 and ∠7

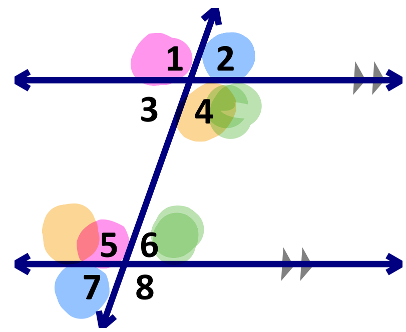


Summary of Angle Pairs Related to Parallel Lines Intersected by a Transversal

set = to each other	add together = 180
These Angle Pairs are Congruent CONGRUENT	**These Angle Pairs are Supplementary** SUPPLEMENTARY
corresponding alternate interior alternate exterior vertical angles	same side interior same side exterior linear pair

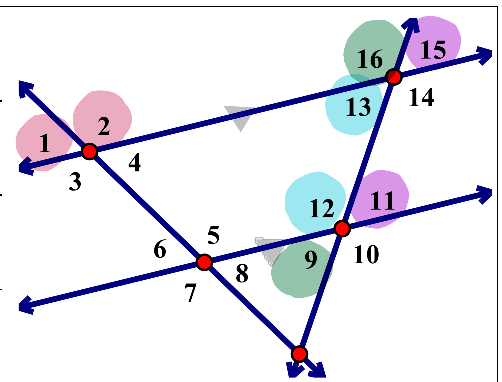
1. Give the name of the relationship of each angle pair for the diagram shown.

∠1 and ∠5	corresponding
∠2 and ∠7	alternate exterior
∠5 and ∠4	alternate interior
∠4 and ∠6	same side interior

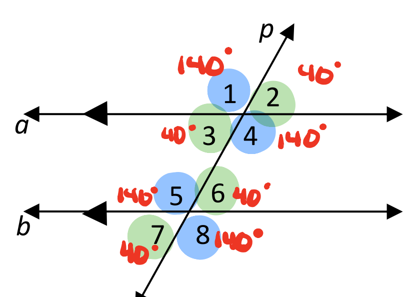


2. Give the name of the relationship of each angle pair for the diagram shown.

∠15 and ∠11	corresponding
∠1 and ∠2	linear pair
∠13 and ∠12	same side interior
∠16 and ∠9	same side exterior



3. Given $a \parallel b$ and p is a transversal. If $m\angle 1 = 140°$, find the measure of each angle giving one reason for each answer.



$m\angle 2 =$ 40°

$m\angle 3 =$ 40°

$m\angle 4 =$ 140°

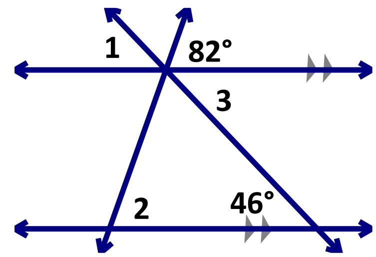
$m\angle 5 =$ 140°

$m\angle 6 =$ 40°

$m\angle 7 =$ 40°

$m\angle 8 =$ 140°

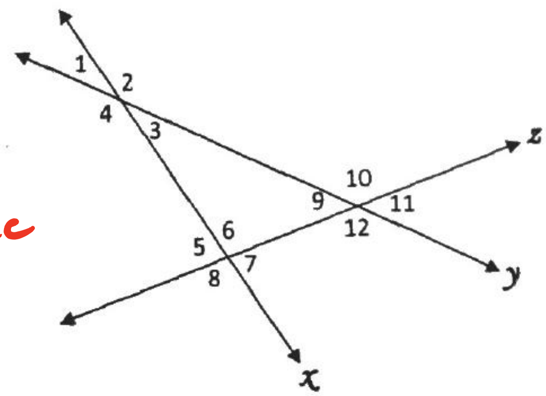
4. Find the measure of each angle and provide the reason for your answer.

m∠2 = $\underline{82°}$	(corresponding)	
m∠3 = $\underline{46°}$	(alternate interior)	
m∠1 = $\underline{46°}$	(vertical to ∠3)	

Geometry
1.6(3) Angle Pairs

Objective: I can use angle pair relationships formed by two parallel lines cut by a transversal to find unknown values or angle measures *(Target 18)*

1. Using the diagram to the right, identify the relationship for the pair of angles.



a) ∠8 and ∠12 corr.

b) ∠12 and ∠1 alt. ext.

c) ∠1 and ∠7 alt. ext.

d) ∠10 and ∠5 same side ext.

e) ∠2 and ∠9 alt. int.

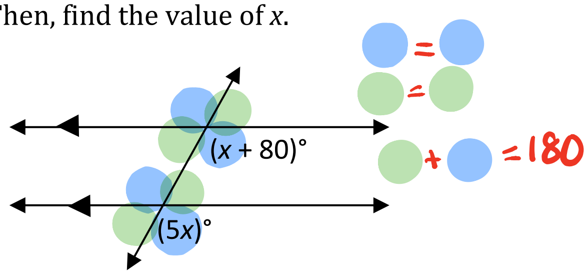
f) ∠4 and ∠5 same side int.

g) ∠3 and ∠9 same side int.

h) ∠5 and ∠1 corr.

2. Identify the relationship of the angles and what they are to each other. Write the equation used to solve for *x*. Then, find the value of *x*.

a)



relationship: corresponding

Congruent or Supplementary?

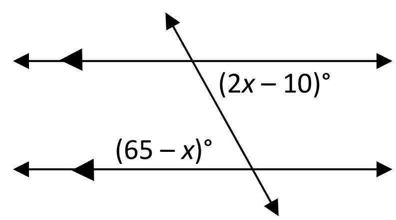
set = to each other

equation: $x + 80 = 5x$

solve:

$\cancel{x} + 80 = 5x$
$\cancel{-x} \quad\quad -x$
$\frac{80}{4} = \frac{\cancel{4}x}{\cancel{4}}$
$\boxed{x = 20}$

b)



relationship: alternate interior

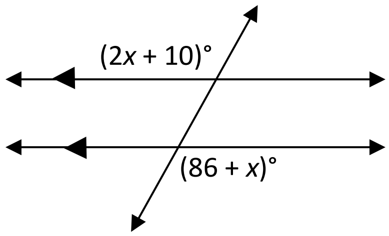
Congruent or Supplementary?

equation: $65 - x = 2x - 10$

solve:

$65 - x = 2x - 10$
$65 = 3x - 10$
$75 = 3x$
$\boxed{x = 25}$

c)



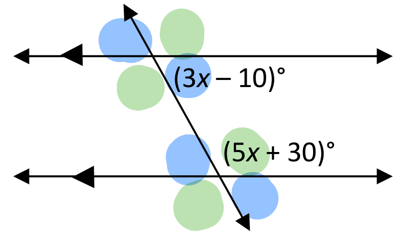
relationship: alternate exterior

Congruent or Supplementary?

equation: $2x+10=86+x$

solve: $\boxed{x=76}$

d)



+ = 180

relationship: same side interior

Congruent or Supplementary? → add them = 180°

equation: $3x-10+5x+30=180$

solve:

$3x-10+5x+30=180$

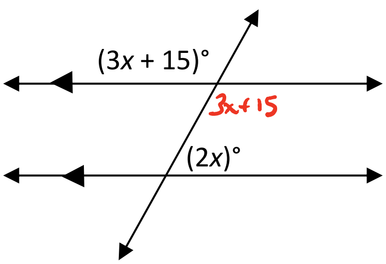
$8x+20=180$

$-20 \quad -20$

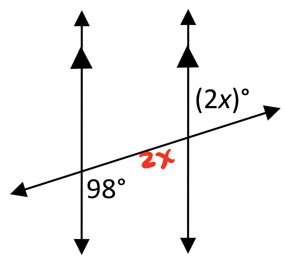
$\frac{8x}{8}=\frac{160}{8}$

$\boxed{x=20}$

e)



f)



relationship: none directly

Congruent or Supplementary?

equation: $3x+15+2x=180$

solve:

$5x+15=180$

$5x=165$

$\boxed{x=33}$

relationship: none directly

Congruent or Supplementary?

equation: $98+2x=180$

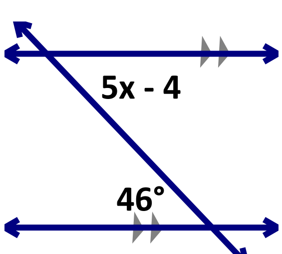
solve:

$2x=82$

$\boxed{x=41}$

3. Solve for the unknown values.

a) x = 10

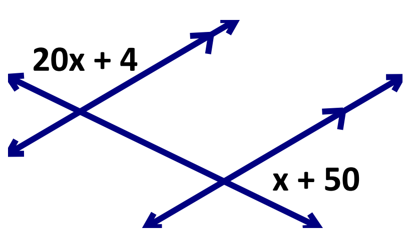


$5x-4=46$

$5x=50$

$\boxed{x=10}$

b) x = 6



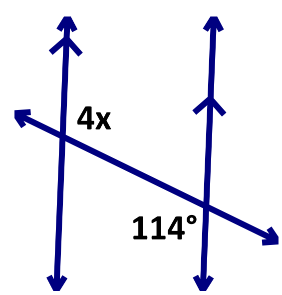
$20x+4+x+50=180$

$21x+54=180$

$21x=126$

$x=6$

c) x = 28.5



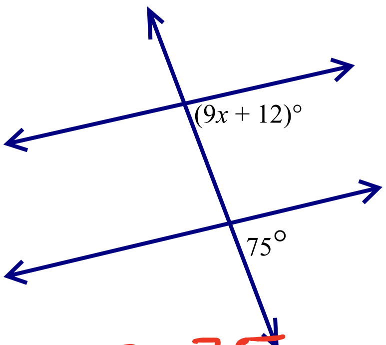
$4x=114$

$\boxed{x=28.5}$

.

4. Solve for the unknown values. Lines that appear parallel are.

a) x = 7

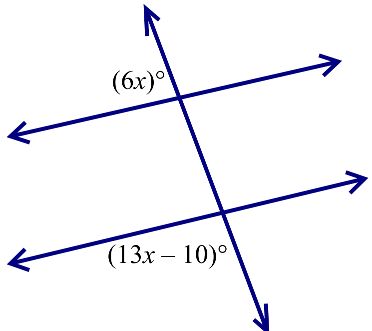


$9x + 12 = 75$

$9x = 63$

$\boxed{x = 7}$

b) x = 10

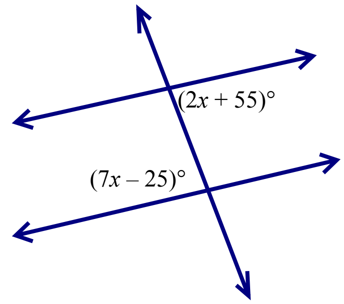


$6x + 13x - 10 = 180$

$19x = 190$

$\boxed{x = 10}$

c) x = 16



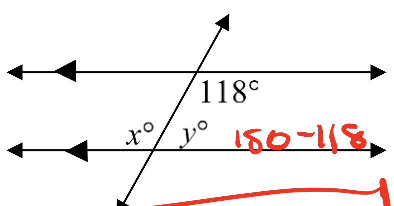
$7x - 25 = 2x + 55$

$5x = 80$

$\boxed{x = 16}$

5. Find the values of *x* and *y*. Put a box around your answer.

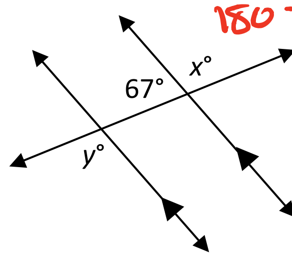
a)



180 − 118

$\boxed{\begin{array}{l} x = 118° \\ y = 62° \end{array}}$

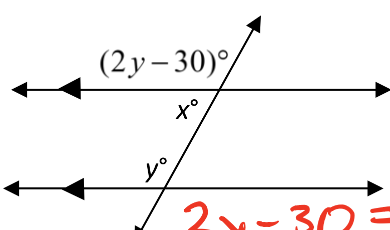
b)



180 − 67

$\boxed{\begin{array}{l} x = 113° \\ y = 113° \end{array}}$

c)

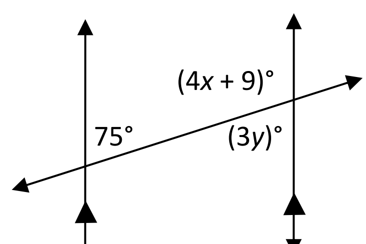


$2y - 30 = y$

$y = 30$

$\boxed{\begin{array}{l} x = 150° \\ y = 30° \end{array}}$

d)



$3y = 75$

$y = 25$

$4x + 9 + 75 = 180$

$4x + 84 = 180$

$4x = 96$

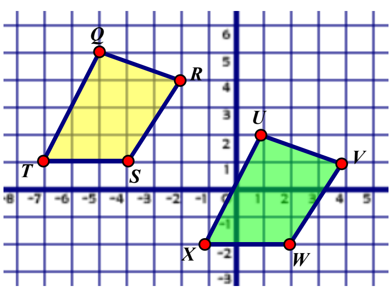
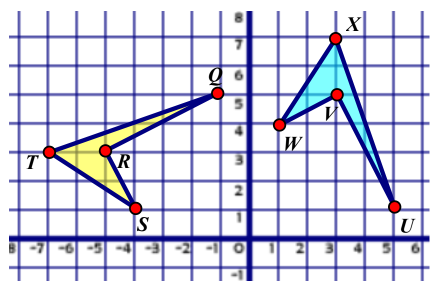
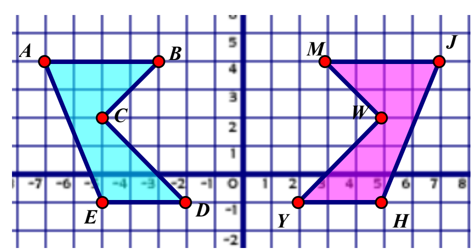
$\boxed{\begin{array}{l} x = 24 \\ y = 25 \end{array}}$

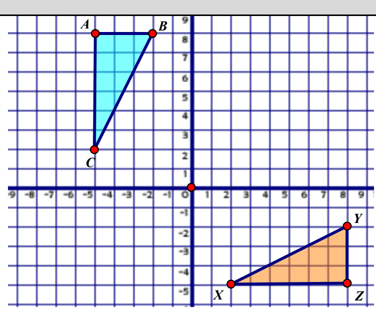
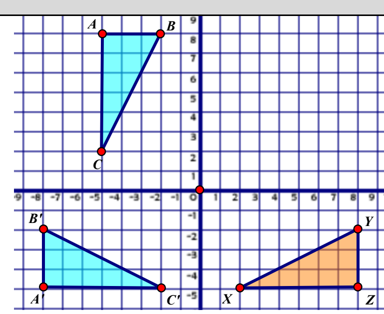
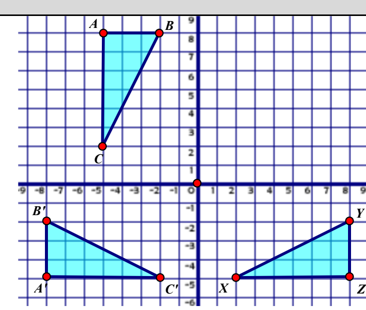
Geometry
1.7 and 1.8(1) Transformations to prove congruency and Congruent Triangles

Objective: I can demonstrate that two figures are congruent by one-to-one mapping using rigid transformations, I can write a congruency statement for congruent figures, and I can identify corresponding parts of congruent figures *(Targets 19-21)*

Review: Isometric transformations (also called rigid motions) are transformations that preserve the size and shape of the pre-image. The isometric transformations are REFLECTION, ROTATION, TRANSLATION.

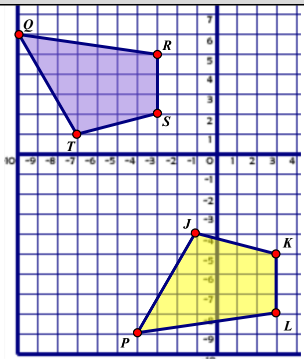
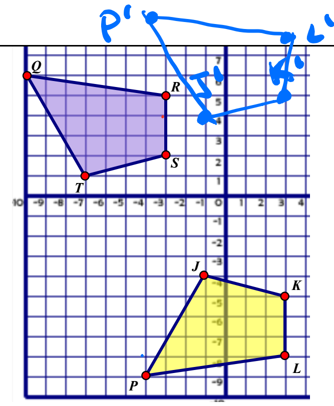
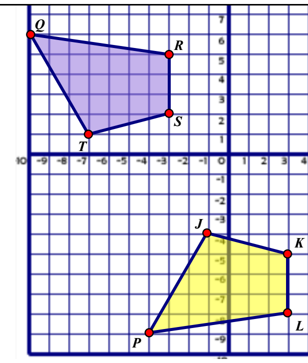
We can use isometric transformations to map one figure onto another to determine congruence.

A translation by <6, -3> maps these two quadrilaterals, so QRST ≅ UVWX	A rotation 270° about the origin maps these two quadrilaterals, so QRST ≅ UVWX	A reflection over the y-axis maps these two pentagons, so ABCDE ≅ JMWYH
		

ΔABC ≅ ΔZYR because I can map ΔABC onto ΔZYR using a rotation and then a reflection.		
Original Relationship	**A 90° rotation about the origin**	**A reflection over the y-axis**
		

$T_{\langle -6,-3\rangle} \circ R_{x\text{-axis}}(QRST)$

1. Is QRST ≅ PLKJ? If so, find the sequence of isometric transformations that map one onto the other.

Original Relationship		
		

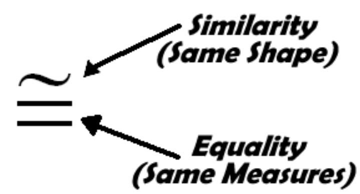
reflect over x-axis

translate by a vector of $\langle -6,-3\rangle$

A **congruence statement** relates the pre-image to the image by identifying the corresponding parts.

Quad ABCD ≅ Quad MNOP

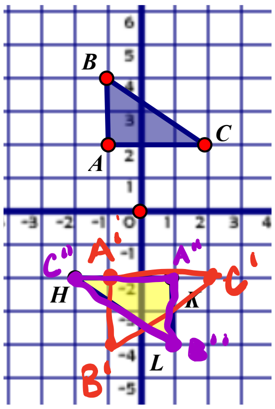
Congruent Corresponding Sides	**Congruent Corresponding Angles**
$\overline{AB} \cong \overline{MN}$	$\angle A \cong \angle M$
$\overline{BC} \cong \overline{NO}$	$\angle B \cong \angle N$
$\overline{CD} \cong \overline{OP}$	$\angle C \cong \angle O$
$\overline{DA} \cong \overline{PM}$	$\angle D \cong \angle P$



2. Complete the following for the examples below:

- Name the transformation(s)
- Complete the congruency statement
- List the pairs of congruent parts (angles and segments)

a)



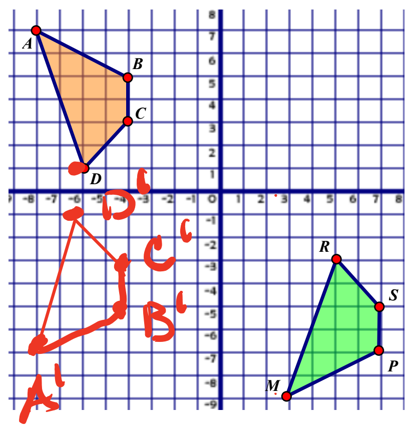
Transformation(s):

$R_{y\text{-axis}} \circ R_{x\text{-axis}} (\Delta ABC)$

$\Delta ABC \cong \Delta KLH$

≅ ∠'s	≅ segments
∠A ≅ ∠K	$\overline{AB} \cong \overline{KL}$
∠B ≅ ∠L	$\overline{BC} \cong \overline{LH}$
∠C ≅ ∠H	$\overline{AC} \cong \overline{KH}$

b)



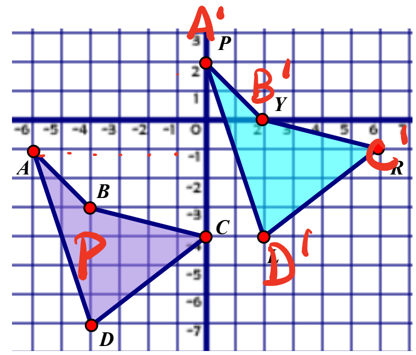
Transformation(s):

$T_{\langle 11,-2\rangle} \circ R_{x\text{-axis}} (RSPM)$

ABCD ≅ MPSR

≅ ∠'s	≅ segments
∠A ≅ ∠M	$\overline{AB} \cong \overline{MP}$
∠B ≅ ∠P	$\overline{BC} \cong \overline{PS}$
∠C ≅ ∠S	$\overline{CD} \cong \overline{SR}$
∠D ≅ ∠R	$\overline{AD} \cong \overline{MR}$

c)



Transformation(s):

$T_{\langle 6,3\rangle} (ABCD)$

ABCD ≅ PYRL

≅ ∠'s	≅ segments
∠A ≅ ∠P	$\overline{AB} \cong \overline{PY}$
∠B ≅ ∠Y	$\overline{BC} \cong \overline{YR}$
∠C ≅ ∠R	$\overline{CD} \cong \overline{RL}$
∠D ≅ ∠L	$\overline{DA} \cong \overline{LP}$

A **congruence statement** for triangles relates one identical object to another by identifying the corresponding parts that match each other.

Determine the congruent sides and angles from the congruence statement below.

ΔABC ≅ ΔDEF

List Congruent Angles	List Congruent Sides
∠A ≅ ∠D	$\overline{AB} \cong \overline{DE}$
∠B ≅ ∠E	$\overline{BC} \cong \overline{EF}$
∠C ≅ ∠F	$\overline{AC} \cong \overline{DF}$

CPCTC – Corresponding Parts of Congruent Triangles are Congruent.

What does this mean in simple terms?

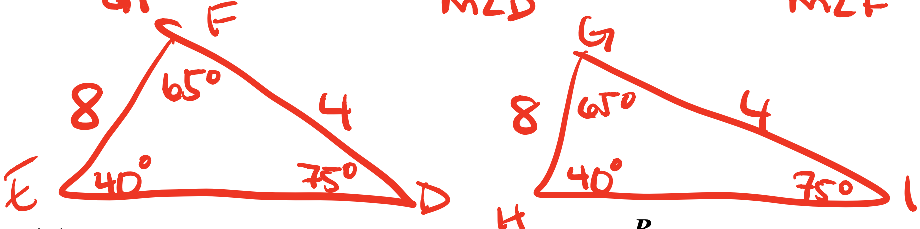
if the Δ's are congruent, all corresponding parts are automatically congruent

3. Use CPCTC to determine all known information about the two triangles: ΔGEO ≅ ΔTRY

$\angle G \cong \angle T$	$\overline{GE} \cong \overline{TR}$
$\angle E \cong \angle R$	$\overline{EO} \cong \overline{RY}$
$\angle O \cong \angle Y$	$\overline{GO} \cong \overline{TY}$

4. Given ΔEFD ≅ ΔHGI with GI = 4, IH = 6, $\boxed{\text{GH}}$ = 8, m∠H = 40°, and m∠D = 75°. Find each.

EF = 8 (HG) FD = 4 (GI) m∠I = 75° (m∠D) m∠G = 65° (m∠F)

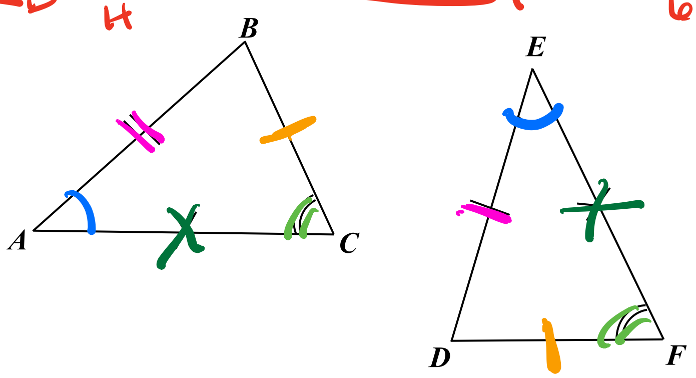


40+75 = 115

180 − 115 = 65

5. The two triangles at the right are congruent.

Identify all corresponding parts



$\angle A \cong \angle E$	$\overline{AB} \cong \overline{ED}$
$\angle C \cong \angle F$	$\overline{AC} \cong \overline{EF}$
$\angle B \cong \angle D$	$\overline{BC} \cong \overline{DF}$

Write a congruence statement: ΔACB ≅ ΔEFD

Write two more congruence statements: ΔFDE ≅ ΔCBA and ΔDFE ≅ ΔBCA

Geometry
1.8(3) Congruent Triangles

Objective: I can determine if two triangles are congruent by SSS, SAS, ASA, AAS, and HL using given markings and assumptions *(Target 22)*

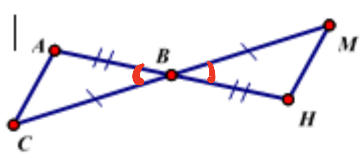
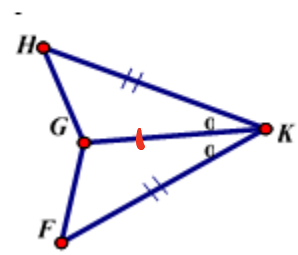
The name of the game is to find the correct combination of corresponding congruent sides and angles that will allow you to prove the triangles congruent.

There are five "shortcut" combinations:

- SSS, or Side-Side-Side;
- SAS, or Side-Angle-Side;
- ASA, or Angle-Side-Angle;
- AAS, or Angle-Angle-Side; and
- HL, or Hypotenuse-Leg.

Most of the time, you must be explicitly given a congruency (told that it exists), or explicitly given a condition that would lead to a congruency (for example, if you are told that a segment is bisected, you can state that the bisected segment is divided into two congruent segments).

The *only* kinds of congruencies that you can "assume", without being told anything at all, are:

Vertical angles	**Reflexive property**
 BOW TIES ARE COOL!	
* all bow ties have vertical angles vertical angles are congruent	any segment or angle is congruent to itself!

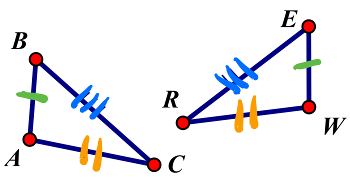
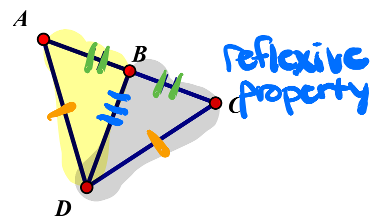
THESE ARE THE ONLY CONDITIONS UNDER WHICH YOU CAN ASSUME THERE IS A CONGRUENCY!

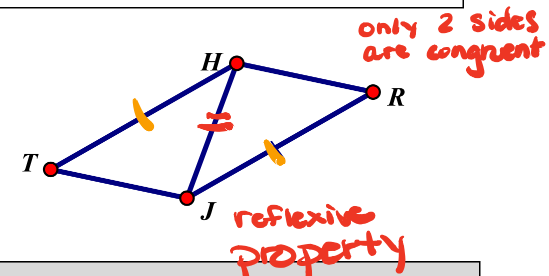
Triangle Congruency Criteria

Side-Side-Side (SSS) Triangle Congruence Postulate: If **three sides** in one triangle are congruent to **three sides** in another triangle, then the triangles are congruent.

1. Determine whether enough information is given to use the SSS Postulate for each diagram.
If yes, write a congruence statement.

YES -OR- NO ΔCBA≅ΔREW	YES -OR- NO ΔDBA≅ΔDBC	YES -OR- NO Δ~~____≅Δ____~~

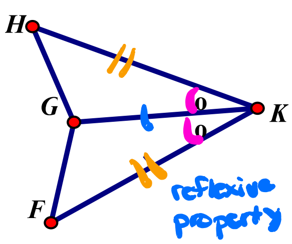



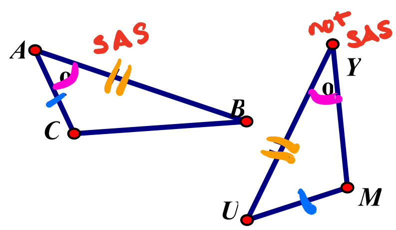


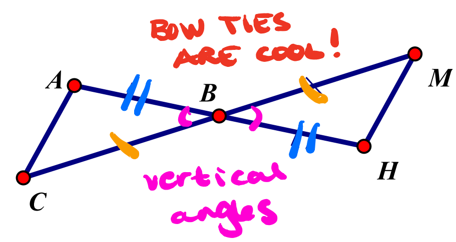
Side-Angle-Side (SAS) Triangle Congruence Postulate: If two sides and the **included** angle in one triangle are congruent to two sides and the **included** angle in another triangle, then the triangles are congruent.

2. Determine whether enough information is given to use the SAS Postulate in each diagram.
If yes, write a congruence statement.

YES -OR- NO ΔHGK≅ΔFGK	YES -OR- NO Δ~~____≅Δ____~~	YES -OR- NO ΔABC≅ΔHBM



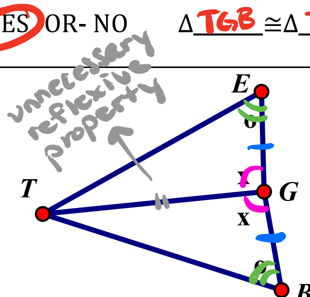




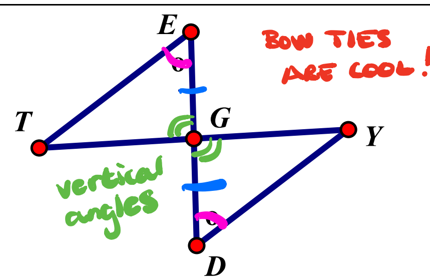
Angle-Side-Angle (ASA) Triangle Congruence Postulate: If two angles and the **included** side in one triangle are congruent to two angles and the **included** side in another triangle, then the triangles are congruent.

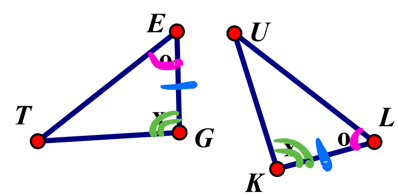
3. Determine whether enough information is given to use the ASA Postulate.
If yes, write a congruence statement.

YES -OR- NO ΔTGB≅ΔTGE	YES -OR- NO ΔTGE≅ΔYGD	YES -OR- NO ΔEGT≅ΔLKU

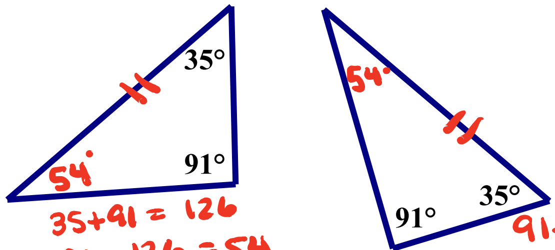


∠TGB ≅ ∠TGE
∠B ≅ ∠E
$\overline{BG}$ ≅ $\overline{EG}$?





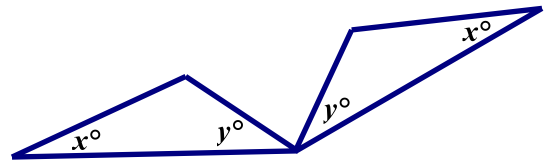
4.a) Find the missing angles in the triangles below.



35+91 = 126
180−126 = 54

91+35=126
180 −126=54

b) Make a conjecture (guess) about the missing angles in theses triangles.



The "THIRD-ANGLE THEOREM" is when two triangles have two pairs of congruent (matching) angles, then the third angles must also be congruent (matching).

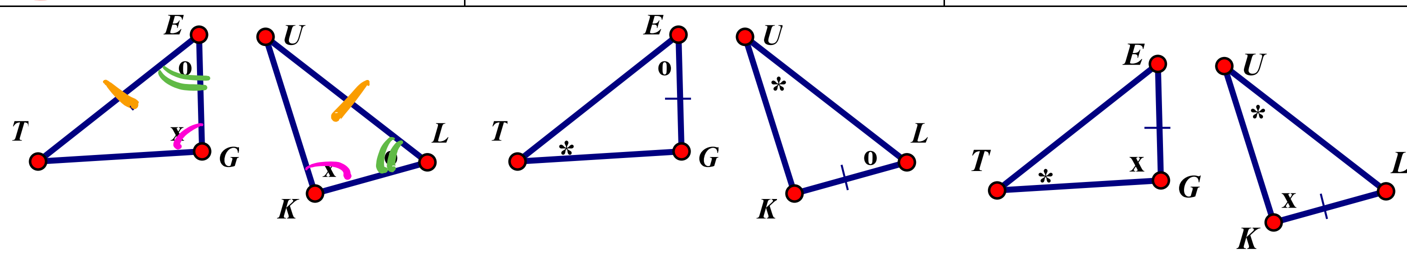
Angle-Angle-Side (AAS) Triangle Congruence Postulate:

If two angles and the non-included side in one triangle are congruent to two angles and the non-included side in another triangle, then the triangles are congruent.

This is a modified version of the ASA Postulate where the third-angle theorem has been applied.

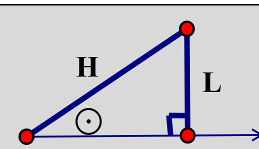
5. Determine whether enough information is given to use the AAS Postulate.
If yes, write a congruence statement.

YES -OR- NO ΔGET ≅Δ KLU	YES -OR- NO ΔEGT ≅Δ LKU	YES -OR- NO ΔGET≅Δ KLU



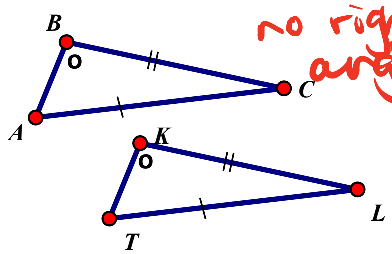
SSA (ASS) is NOT a congruence postulate because it can result in two <u>different</u> triangles with the same information. However, there is a case of SSA (ASS) we CAN use as a congruence postulate.

This case is known as HL, or <u>Hypotenuse</u> (H) – <u>Leg</u> (L). It gets this special name because it is a right triangle. In a right triangle if you know two sides, you can use the Pythagorean Theorem to calculate the third side. Now you have SSS or SAS. **HL forms a triangle congruence relationship.**

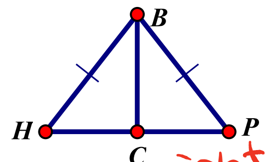


6. Determine whether enough information is given to use **HL**. If yes, write the congruence statement.

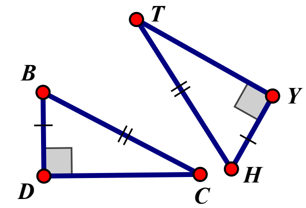
YES -OR- NO Δ~~_____≅Δ_____~~	YES -OR- NO Δ_____≅Δ_____	YES -OR- NO ΔBDC≅ΔHYT



no right angles

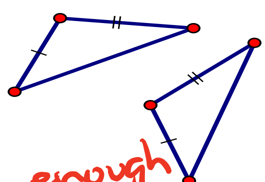


no right angles

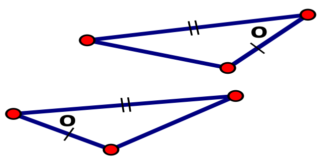


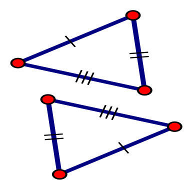
7. Are the following pairs of triangles congruent?
If they are, name their congruence criteria (SSS, SAS, ASA, AAS, or HL)

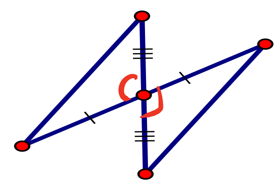
a) YES -OR- NO ~~______~~ b) YES -OR- NO SAS c) YES -OR- NO SSS c) YES -OR- NO SAS



not enough information







BOWTIES ARE COOL!

8. Are the following pairs of triangles congruent? If yes, create a congruent statement and name the congruence criteria (SSS, SAS, ASA, AAS, or HL).

a) Yes -OR- No

ΔDEC ≅ ΔNMC

Criteria AAS

b) Yes -OR- No

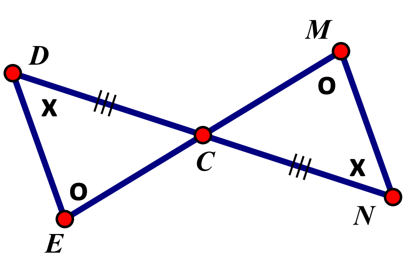
ΔDCE ≅ ΔRCE

Criteria ASA

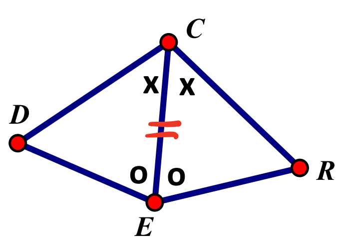
c) Yes -OR- No

ΔFCD ≅ ΔCFT

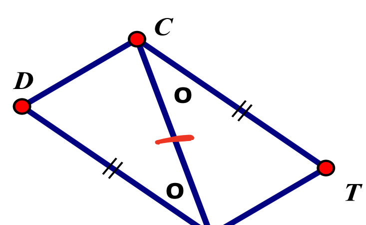
Criteria SAS



(or ASA, if using the vertical angles)



reflexive property

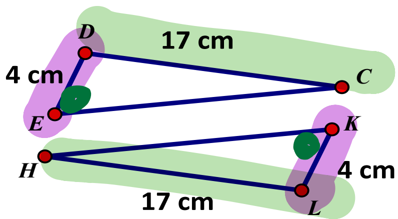


reflexive property

d) Yes -OR- No

Δ______ ≅ Δ______

Criteria __________

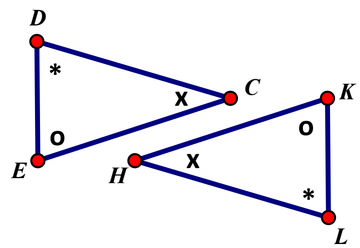


angle is not where the sides meet

e) Yes -OR- No

Δ______ ≅ Δ______

Criteria __________



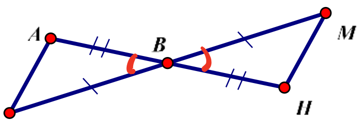
AAA is not a thing for congruency

Geometry
1.8(4) Congruent Triangles

Objective: I can prove two triangles are congruent by SSS, SAS, ASA, AAS, and HL using givens, assumptions, and theorems *(Target 23)*

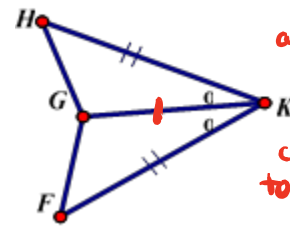
REMEMBER: The only things you can ever assume are:

Vertical angles



BOW TIES ARE COOL!

Reflexive property



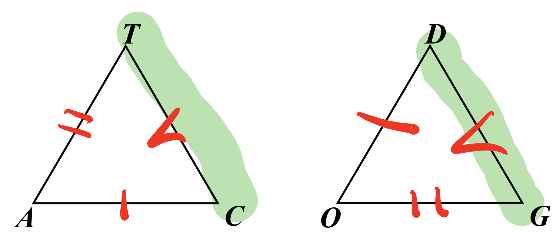
a segment or angle is always congruent to itself

Congruence Criteria and Proof: SSS and SAS

Side-Side-Side (SSS) Triangle Congruence Postulate: If three sides in one triangle are congruent to three sides in another triangle, then the triangles are congruent. **Side-Angle-Side (SAS) Triangle Congruence Postulate:** If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the triangles are congruent.

1. Based on the given information, what additional information is needed in order to prove the triangles congruent by the SSS Postulate? $\overline{TC} \cong \overline{GD}$

Given: $\overline{CA} \cong \overline{DO}$ and $\overline{TA} \cong \overline{OG}$



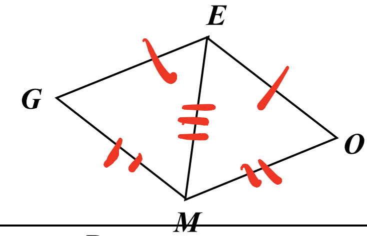
Write a congruence statement.

Δ CAT $\cong \Delta$ DOG

SSS SAS ASA AAS HL

2. Given: $\overline{GE} \cong \overline{EO}$ and $\overline{GM} \cong \overline{OM}$ FIRST

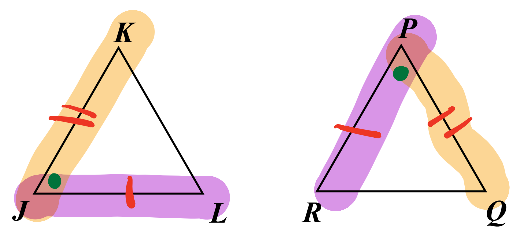
Prove: $\Delta GEM \cong \Delta OEM$ last



Statements	Reasons
1. $\overline{GE} \cong \overline{OE}$ and $\overline{GM} \cong \overline{OM}$	1. Given
2. $\overline{EM} \cong \overline{EM}$	2. reflexive property
3. $\Delta GEM \cong \Delta OEM$	3. SSS

3. Based on the given information, what additional information is needed in order to prove the triangles congruent by the SAS Postulate? $\angle J \cong \angle P$

Given: $\overline{JL} \cong \overline{PR}$ and $\overline{JK} \cong \overline{PQ}$



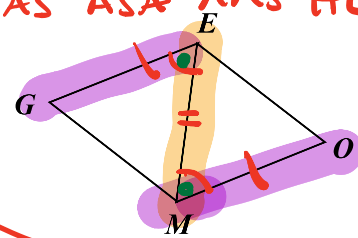
Write a congruence statement.

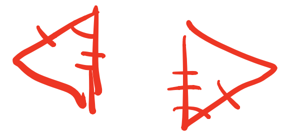
Δ JKL $\cong \Delta$ PQR

SSS SAS ASA AAS HL

4. Given: $\overline{GE} \cong \overline{OM}$ and $\angle GEM \cong \angle OME$

Prove: $\Delta GEM \cong \Delta OME$





Statements	Reasons
1. $\overline{GE} \cong \overline{OM}$ and $\angle GEM \cong \angle OME$	1. Given
2. $\overline{EM} \cong \overline{EM}$	2. reflexive property
3. $\Delta GEM \cong \Delta OME$	3. SAS

Congruence Criteria and Proof: ASA, AAS, and HL

Angle-Side-Angle (ASA) Triangle Congruence Postulate: If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the triangles are congruent.

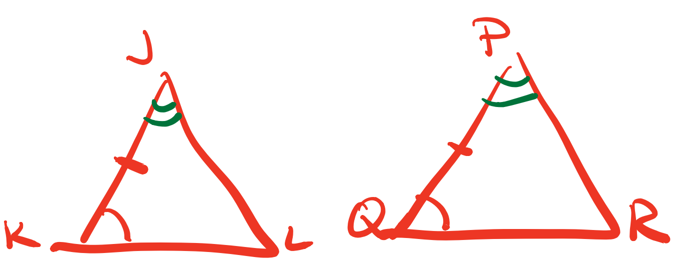
Angle-Angle-Side (AAS) Triangle Congruence Postulate: If two angles and the non-included side in one triangle are congruent to two angles and the non-included side in another triangle, then the triangles are congruent.

Hypotenuse-Leg (HL). In a right triangle if you know a leg and the hypotenuse are congruent to the leg and hypotenuse in another triangle, then the right triangles are congruent.

5. What <u>additional</u> information is needed?

Given: $\overline{JK} \cong \overline{PQ}$ and $\angle K \cong \angle Q$

Prove: $\Delta JKL \cong \Delta PQR$ by ASA



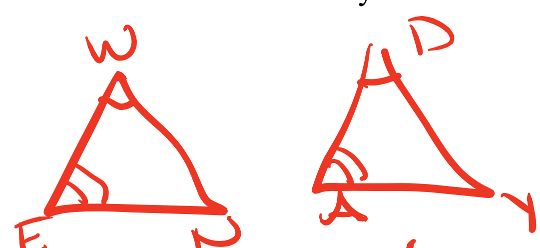
ASA

$\boxed{\angle J \cong \angle P}$

6. What <u>additional</u> information is needed?

Given: $\angle W \cong \angle D$ and $\angle E \cong \angle A$

Prove: $\Delta WEN \cong \Delta DAY$ by ASA

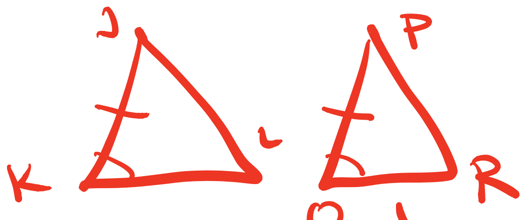


$\boxed{\overline{WE} \cong \overline{DA}}$

7. What additional information is needed?

Given: $\overline{JK} \cong \overline{PQ}$ and $\angle K \cong \angle Q$

Prove: $\Delta JKL \cong \Delta PQR$ by AAS



$\boxed{\angle L \cong \angle R}$

8. What additional information is needed?

Given: $\overline{JK} \cong \overline{PQ}$ and $\angle K$, $\angle Q$ right angles

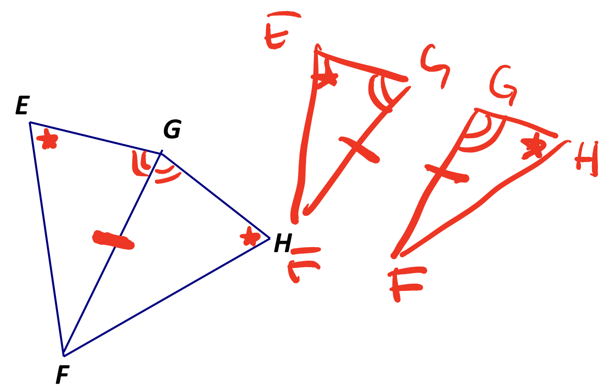
Prove: $\Delta JKL \cong \Delta PQR$ by HL

$\boxed{\overline{JL} \cong \overline{PR}}$

9. **Given**: $\angle E \cong \angle H$

$\overrightarrow{GF}$ bisects $\angle EGH$

Prove: $\Delta EGF \cong \Delta HGF$

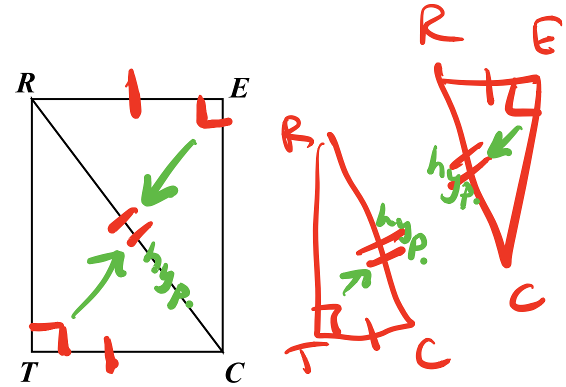


Statements	Reasons
1. $\angle E \cong \angle H$	1. Given
2. $\overrightarrow{GF}$ bisects $\angle EGH$	2. Given
3. $\angle EGF \cong \angle HGF$	3. def. of $\angle$ bisector
4. $\overline{GF} \cong \overline{GF}$	4. reflexive property
5. $\Delta EGF \cong \Delta HGF$	5. SAA

10. **Given**: $\overline{TC} \cong \overline{RE}$

$\angle T$ and $\angle E$ are right angles

Prove: $\Delta TRC \cong \Delta ECR$



	Statements	**Reasons**
leg	① $\overline{TC} \cong \overline{RE}$	① Given
rt∠'s	② ∠T and ∠E are rt.∠'s	② Given
hyp	③ $\overline{RC} \cong \overline{RC}$	③ reflexive property
	④ $\Delta TRC \cong \Delta ECR$	④ HL

Geometry
1.8(5) and 1.8(6) Congruent Triangles

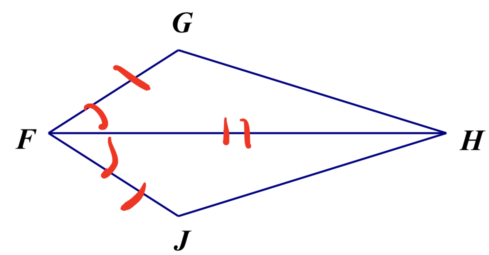
Objective: I can prove two triangles are congruent using givens, assumptions, and theorems, and I can use CPCTC in a proof *(Targets 23-24)*

CPCTC is the reason that we can identify congruent sides and angles from a triangle congruence statement. CPCTC stands for <u>C</u>orresponding <u>P</u>arts of <u>C</u>ongruent <u>T</u>riangles are <u>C</u>ongruent.

In the proofs below, the 'PROVE' item is NOT to prove two triangles congruent*…. It is actually to prove two corresponding pieces (angles or sides) to be congruent. The general strategy will be to <u>first prove triangles to be congruent</u> so that we can make a statement about sides or angles also being congruent.*

1. **Given**: $\angle GFH \cong \angle JFH$
$\overline{GF} \cong \overline{JF}$

Prove: Δ GFH ≅ Δ JFH by SAS



How do we know that $\overline{GH} \cong \overline{JH}$?

because if the Δs are ≅, then all their corresponding parts are ≅

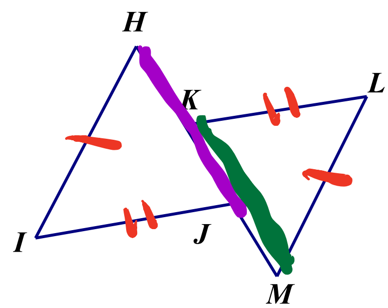
What other conclusions can we make?

∠G ≅ ∠J
∠GHF ≅ ∠JHF

2. Given: $\overline{HJ} \cong \overline{KM}$
$\overline{IH} \cong \overline{LM}$
$\overline{IJ} \cong \overline{LK}$

Prove: $\angle H \cong \angle M$

CPCTC

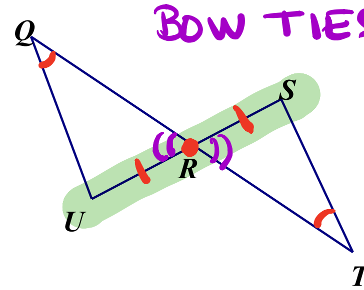


Statements	Reasons
1. $\overline{HJ} \cong \overline{KM}$	1. Given
2. $\overline{IH} \cong \overline{LM}$	2. Given
3. $\overline{IJ} \cong \overline{LK}$	3. Given
4. Δ HIJ ≅ Δ MLK	4. SSS
5. ∠H ≅ ∠M	5. CPCTC

3. Given: R is the midpoint of $\overline{SU}$

$\angle Q \cong \angle T$

Prove: $\overline{QR} \cong \overline{RT}$ CPCTC



BOW TIES ARE COOL!

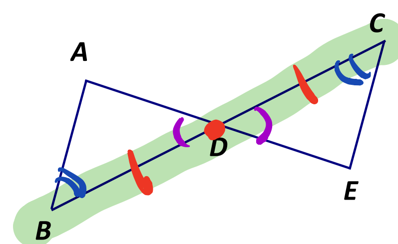
Statements	Reasons
1 R is midpoint $\overline{SU}$	1 Given
2 $\angle Q \cong \angle T$	2 Given
3 $\overline{UR} \cong \overline{SR}$	3 def. of midpoint
4 $\angle QRU \cong \angle TRS$	4 vertical angles $\cong$
5 $\Delta QRU \cong \Delta TRS$	5 AAS
6 $\overline{QR} \cong \overline{RT}$	6 CPCTC

More Practice with proofs

4. Given: D is the midpoint of $\overline{BC}$

$\overline{AB} \parallel \overline{EC}$ parallel

Prove: $\Delta ADB \cong \Delta EDC$ don't need CPCTC



BOW TIES ARE COOL!

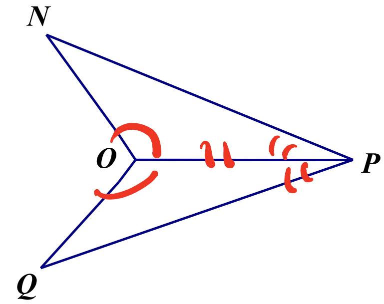
Statements	**Reasons**
1. D is midpoint $\overline{BC}$	1. Given
2. $\overline{AB} \parallel \overline{EC}$	2. Given
3. $\angle ADB \cong \angle EDC$	3. vertical $\angle$'s $\cong$
4. $\overline{BD} \cong \overline{CD}$	4. def. of midpoint
5. $\angle B \cong \angle C$	5. alt. int. $\angle$'s $\cong$

$\Delta ADB \cong \Delta EDC$ ASA

5. Given: $\overrightarrow{PO}$ bisects $\angle NPQ$

$\angle NOP \cong \angle QOP$

Prove: $\overline{ON} \cong \overline{OQ}$ CPCTC

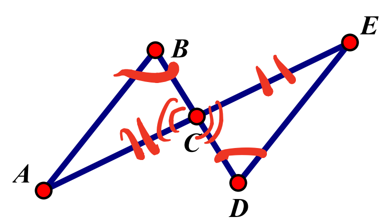


Statements	Reasons
1 $\overrightarrow{PO}$ bisects $\angle NPQ$	1 Given
2 $\angle NOP \cong \angle QOP$	2 Given
3 $\angle NPO \cong \angle QPO$	3 def. of $\angle$ bisector
4 $\overline{OP} \cong \overline{OP}$	4 reflexive property
5 $\triangle NOP \cong \triangle QOP$	5 ASA
6 $\overline{ON} \cong \overline{OQ}$	6 CPCTC

6. Given: $\angle B \cong \angle D$

$\overline{AC} \cong \overline{EC}$

Prove: $\angle A \cong \angle E$ CPCTC



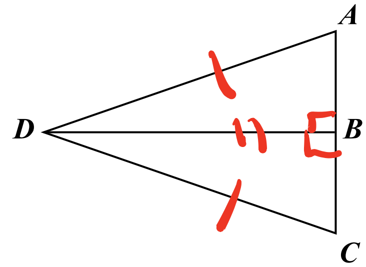
Statements	Reasons
1 $\angle B \cong \angle D$	1 Given
2 $\overline{AC} \cong \overline{EC}$	2 Given
3 $\angle BCA \cong \angle DCE$	3 vertical $\angle$'s $\cong$
4 $\triangle BCA \cong \triangle DCE$	4 AAS
5 $\angle A \cong \angle E$	5 CPCTC

7. Given: $\overline{DA} \cong \overline{DC}$

$\overline{DB} \perp \overline{AC}$ perpendicular

Prove: $\Delta ADB \cong \Delta CDB$

no CPCTC

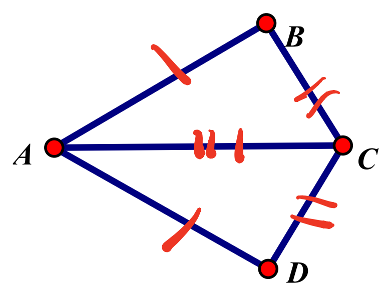


Statements	Reasons
1. $\overline{DA} \cong \overline{DC}$, $\overline{DB} \perp \overline{AC}$	1. Given
2. $\angle DBA$ and $\angle DBC$ are right angles	2. def. of $\perp$ lines
3. $\overline{DB} \cong \overline{DB}$	3. reflexive property
4. $\Delta ADB \cong \Delta CDB$	4. HL

8. Given: $\overline{AB} \cong \overline{AD}$

$\overline{BC} \cong \overline{DC}$

Prove: $\angle BAC \cong \angle DAC$ CPCTC



Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{BC} \cong \overline{DC}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. reflexive property
4. $\Delta ABC \cong \Delta ADC$	4. SSS
5. $\angle BAC \cong \angle DAC$	5. CPCTC

Geometry
1.9 Triangle Properties

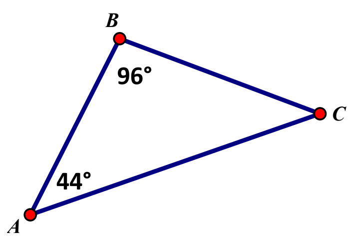
Objective: I can solve for unknowns by using: the sum of the interior angles is 180°; the base angles of an isosceles triangle are congruent; the exterior triangle theorem; and the triangle midsegment theorem *(Targets 25-28)*

The sum of the interior angles of a triangle is 180°
(Triangle Sum Theorem)

Animation of Informal Proof: https://www.youtube.com/watch?v=KwProyEPRgE

1. Determine the measure of the angle.

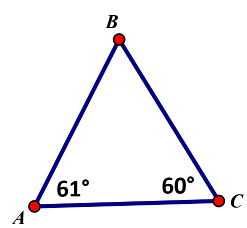
a) m∠C = 40



$96 + 44 = 140$

$180 - 140 = 40$

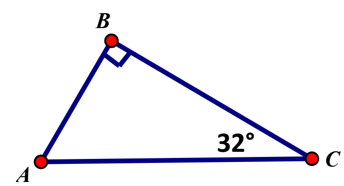
b) m∠B = 59°



$61 + 60 = 121$

$180 - 121 = 59$

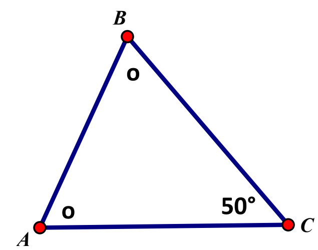
c) m∠A = 58°



$90 + 32 = 122$

$180 - 122 = 58$

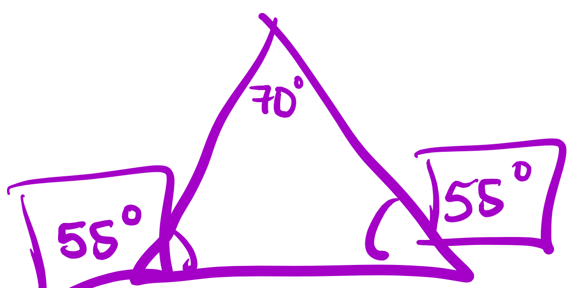
d) m∠A = 65°



$180 - 50 = 130$

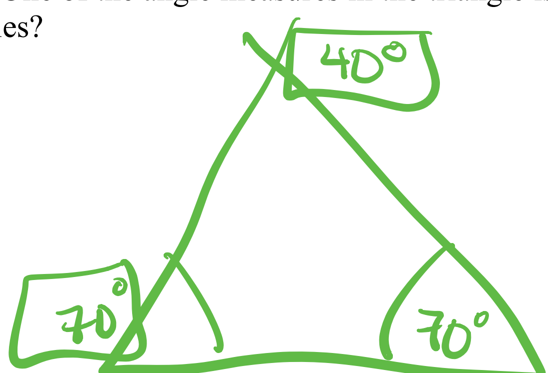
$130 \div 2 = 65$

2. Two of the angles in a triangle are congruent. One of the angle measures in the triangle is 70°. What are the possible measures of the other two angles?



$180 - 70 = 110$

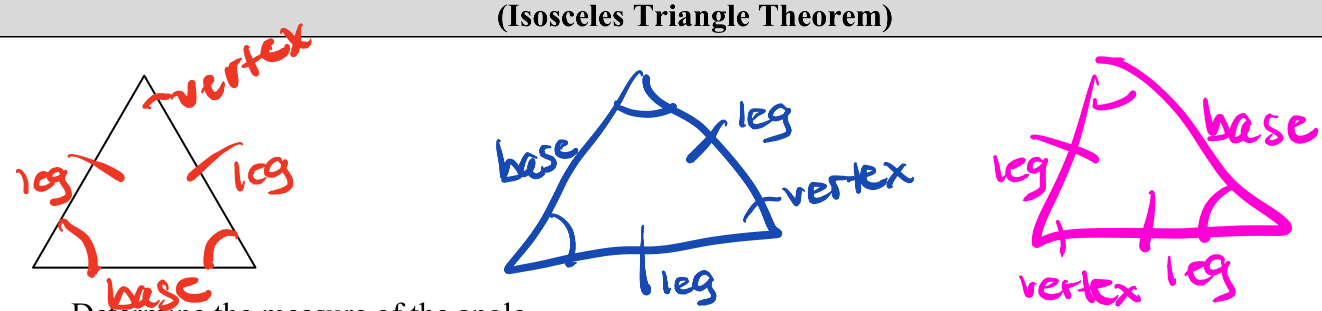
$\frac{110}{2}$



$70 + 70 = 140$

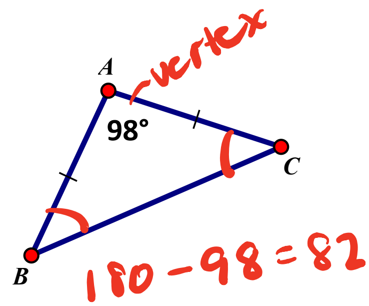
$180 - 140$

The Base Angles of an Isosceles are Equal
(Isosceles Triangle Theorem)



3. Determine the measure of the angle.

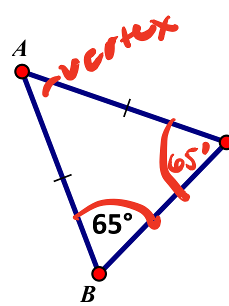
a) m∠C = 41°



180 − 98 = 82

$\frac{82}{2}$

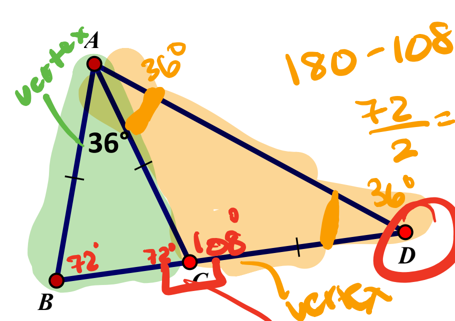
b) m∠A = 50°



65 + 65 = 130

180 − 130

c) m∠D = 36°



180 − 108 = 72

$\frac{72}{2} = 36$

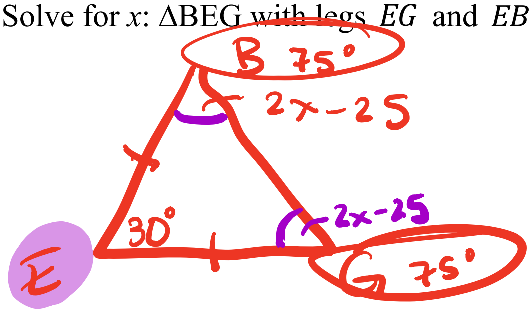
180 − 36 = 144

$\frac{144}{2}$

linear pair

180 − 72 = 108

4. Solve for x: ΔBEG with legs $\overline{EG}$ and $\overline{EB}$, $m\angle B = (2x - 25)°$ and $m\angle E = 30°$.



180 − 30 = 150

$\frac{150}{2} = 75$

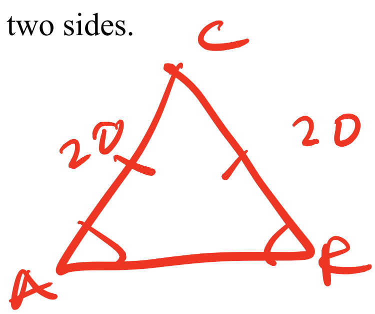
2x − 25 = 75
+25 +25
2x = 100

$\boxed{x = 50}$

The converse of the Isosceles Triangle is also true, meaning that if two angles of a triangle are congruent, the triangle must have two congruent sides (opposite from those angles).

→ congruent ∠'s

5. ΔCAR has base angles ∠A and ∠R. The perimeter of ΔCAR is 44 with CA = 20. Find the lengths of the other two sides.



CR = 20
because it is ≅ $\overline{CA}$

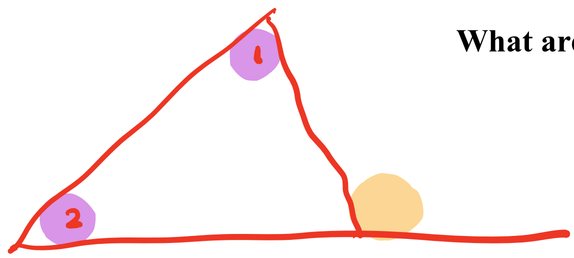
20 + 20 = 40

44 − 40 = 4

AR = 4

Each exterior angle of a triangle is equal to the sum of its two remote interior angles.
(Exterior Angle Theorem)

What is an exterior angle?



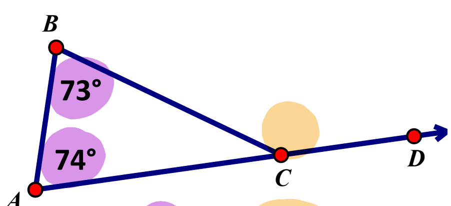
What are its remote interior angles?

1 + 2 =

Animation of informal proof: https://www.youtube.com/watch?v=NpVsF4St6eI

6. Determine the missing information.

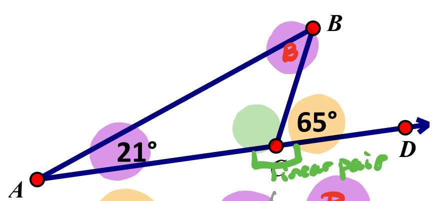
a) m∠BCD = 147°



73 + 74 = 147

b) m∠B = 44°

m∠BCA = 115°

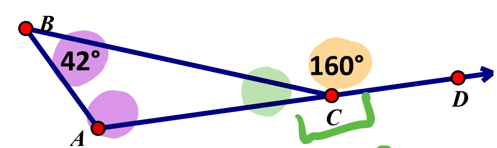


65 = 21 + B
−21 −21
m∠B = 44

180 − 65 = 115°
linear pair

c) m∠A = 118°

m∠BCA = 20°

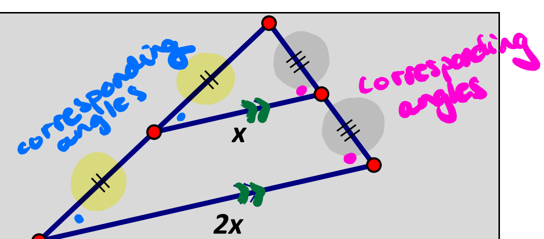


linear pair

180 − 160 = 20

160 − 42 = 118

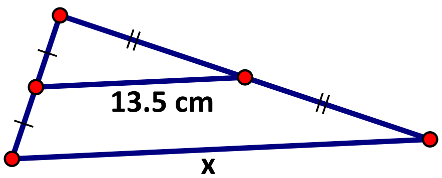
The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.
(Midsegment Theorem)



Video of Informal Proof: https://www.youtube.com/watch?v=nF_Ltl4YSsI

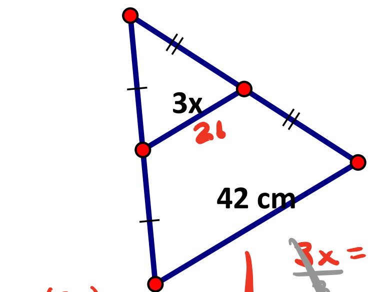
7. Determine the missing information.

a) x = 27



$2(13.5) = x$

b) x = 7



$2(3x) = 42$

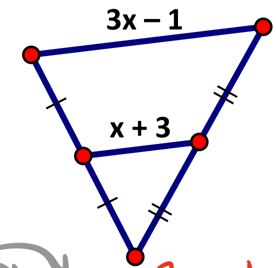
$\frac{6x}{6} = \frac{42}{6}$

$x = 7$

$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

c) x = 7



$2(x+3) = 3x - 1$

$2x + 6 = 3x - 1$

$-2x \quad -2x$

$6 = x - 1$

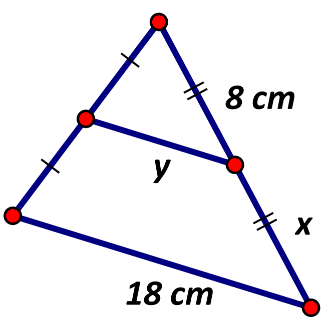
$+1 \quad +1$

$x = 7$

8. Δ*PQR* has side lengths 20, 18, and 24.
Given that X, Y, and Z are the midpoints of the sides of Δ*PQR*, find the perimeter of ΔXYZ.

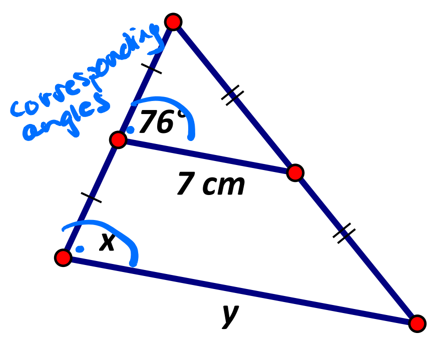
9. Find the missing values:

a)



x = 8 cm y = 9 cm

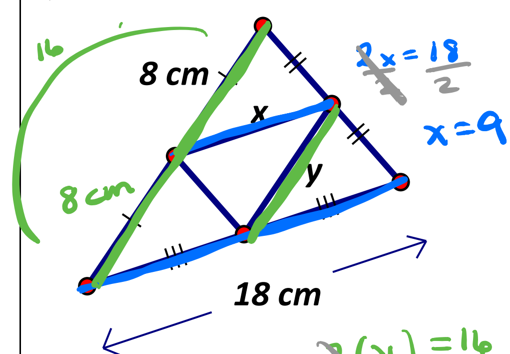
b)



$2(7) = y$

x = 76 ° y = 14 cm

c)



$\frac{2x}{2} = \frac{18}{2}$

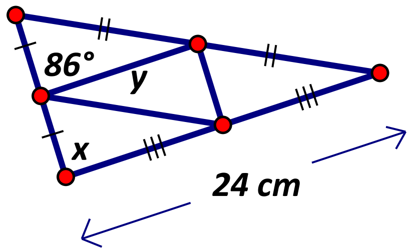
$x = 9$

$\frac{2(y)}{2} = \frac{16}{2}$

$y = 8$

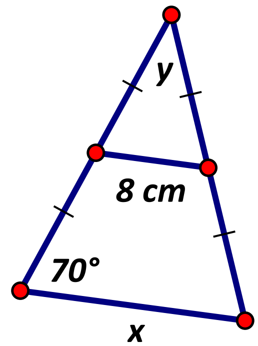
x = 9 cm y = 8 cm

d)



x = 86 ° y = 12 cm

e)



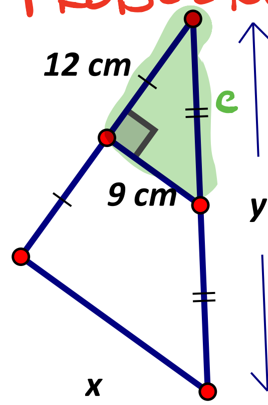
x = 16 cm y = 40 °

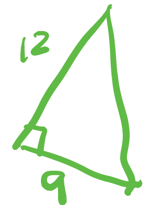
70+70=140

180-140=

40

f) CHALLENGE PROBLEM!





$9^2+12^2=c^2$

$81+144=c^2$

$\sqrt{225}=\sqrt{c^2}$

$c=15$

x = 18 cm y = 30 cm

2(15)